

Chiral SUSY Theories with a Suppressed SUSY Charge

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Abstract

The well-known Chiral and Gauge SUSY Actions realize the SUSY charge in terms of transformations among the Fields. These transformations are included in the Master Equation by coupling them to Sources. Here we show that there are new local SUSY Actions where the Chiral SUSY transformations are realized in terms of transformations among both Fields and Sources. These Actions can be easily obtained from the Chiral case by a very simple and local ‘Exchange Transformation’, which carries along all the interactions without difficulty. For these new SUSY Actions, the SUSY charge does not exist in the relevant sector, because Sources do not satisfy Equations of Motion.

Nevertheless, the ‘Exchange Transformation’ ensures that the new Master Equation is true for the new Action. As a consequence, the Master Equation also is true for the new 1PI Generating Functional. This implies that a ‘Suppressed SUSY Charge’ version of SUSY is still present. SUSY certainly becomes more obscure and less constrained in this case. But it is still very restrictive.

The new theories can be obtained from the old theories by using a special technique, but it is not true that they are a sort of ‘broken version of supersymmetry’. They are simply a new type of theory that is governed by Supersymmetry, but without the use of Supercharges (except perhaps in some sectors).

In particular the number of physical Bosonic and Fermionic degrees of Freedom are not equal for these new (sub)-Actions, although there is still Boson/Fermion mass degeneracy in a (sub)-Action, so long as there is still a Boson present. Notably, there is even a SUSY (sub)-Action where the physical Scalars are not present, so that the (sub)-Action contains physical Fermions only. In this theory the degeneracy of Bosonic and Fermionic masses is obviously not present, and this happens without the spontaneous or explicit breaking of SUSY. No such ‘Exchange Transformation’ has been found for SUSY gauge Actions.

1. Introduction: In spite of a large amount of work, there is still much that is unknown about SUSY [1,2,3,4,5,6]. Even its representation theory is filled with unanswered questions [7]. Much recent work has concerned itself with geometric and duality issues in supersymmetric theories. Some of this is associated with Branes, M theory and AdS CFT [8,9,10,11]. A large body of SUSY work has, understandably, focussed on phenomenology and experimental signals, while leaving the problems of the origin of the spectrum of mass splitting for future work [12,13,14]. The progress reported in this paper is the direct result of a study of the algebraic problem of the local BRST cohomology of the Chiral Multiplet [15,16,17], which examined the solution to the tachyon problem. Previous work, troubled by the tachyon problem, was in [18,19,20,21,22,23]

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using some of the methods in [24,25]. All of that work was based on the Master Equation formulation of symmetries [26,27,28,29,30,31].

From the beginning of research into SUSY, it has been noticed by many authors that SUSY seems to hint at solutions to various problems. But these hints then turn to disappointment, because the effort to remove the mass degeneracy of the supermultiplets, using spontaneous SUSY breaking, tends to spoil the nice properties of the theory [32]. This has led some authors to wonder whether the mass degeneracy of SUSY can be removed, even when SUSY itself is not really spontaneously (or explicitly) broken at all [33].

The theory presented here shows how the mass degeneracy can be removed for Chiral Multiplets, without spontaneous or explicit breaking of SUSY. The result is a sort of compromise between the usual SUSY theories that have a conserved SUSY charge, and theories which have no SUSY at all. We use an Exchange Transformation² to change the original normal SUSY theory to a theory with a non-conserved, but still very relevant, SUSY charge. The new theory satisfies a Master Equation that is very similar to the Master Equation of the parent SUSY theory. The method here does not apply to SUSY Gauge theory so as to remove its SUSY charge. But the presence of SUSY Gauge theory is not a problem for the method.

2. Half-Chiral Multiplets and Un-Chiral Multiplets: For each Chiral Multiplet in an Action, there is a choice to be made. One can simply leave it as a Chiral Multiplet, or one can use an Exchange Transformation to transform it to a new kind of Multiplet, with a new Action and new physics. This Exchange Transformation transforms some or all of the Scalar Field A to a Zinn Source³ J and the Zinn Source Γ to an Antighost Field η . The Zinn Source Γ is the Source for variations of the Scalar Field A (in the old Action). The Zinn Source J is the Source for variations of the Antighost Field η (in the new Action).

The Exchange Transformations of the Master Equation that we will introduce in this paper will be said to give rise to a **Half-Chiral Multiplet**, when one Scalar remains, and to an **Un-Chiral Multiplet**, when no Scalars remain. This decision can be made for each Chiral Multiplet in the theory, separately. Since the two Scalars in a Chiral Multiplet are not equivalent to each other (they differ in parity for example), there are really three different Exchange Transformations possible for each Chiral Multiplet. The Exchange Transformation is, in terms of its construction, a sort of Canonical Transformation, except that it takes one theory to a different theory for the Master Equation case. This is explained in Section 15 below.

The Half-Chiral Multiplets are useful for the spontaneous breaking of Gauge symmetry. However, some of the mass degeneracy survives for the Gauge/Higgs sector, as will be shown in [34]. The Un-Chiral Multiplets are useful to make an Action with no Scalars, and we will use them for the SSM in [34], to eliminate all the Squarks and Sleptons, leaving no mass degeneracy in that Matter sector. Clearly this is a form of ‘SUSY Charge Suppression’, which eliminates the SUSY charge in a particular sector, while preserving the Master Equation, which has a strong influence from SUSY.

A pleasant feature is that the Exchange Transformation also allows us to construct interactions for these new ‘Suppressed SUSY’ theories. These follow directly from the initial Action, which is made entirely from Gauge and Chiral Multiplets (and ultimately Supergravity). The Exchange Transformation also allows one to show, in a formal way, that, in perturbation theory, one can construct new Quantum Field theories from the new theories. These preserve SUSY with the new Master Equation, as is shown in Sections 23 and 24 for the Un-Chiral Multiplet case. A similar derivation can be done for the Half-Chiral Multiplet.

²This is like a canonical transformation—it ensures that the new Master Equation yields zero with the new Action. See Section 15 below.

³The original formulation of the Master Equation as an ‘antibracket’ was by Zinn-Justin [26,27,28,29,30,31]. The Sources in the Master Equation are sometimes called ‘Antifields’, following [35]. The author thinks that this term is misleading and confusing. The so-called ‘Antifields’ are very definitely not Fields—they are Sources. The term ‘anti’ is also confusing, because the term ‘Antifield’ would naturally mean the ‘Field that creates Antiparticles’. So here these Sources are called Zinn Sources. The Zinn Sources do not get integrated in the Feynman path integral, whereas Fields do get integrated, and so, of course, do their Complex Conjugates, the Antifields. We use the term ‘Zinn Action’ to denote the part of the Action that is at least linear in Zinn Sources. It is unphysical, but useful to keep track of the symmetry.

3. This paper is organized as follows: We introduced some names for the new theories in Section 2. Next, in Section 4, we will pose and answer some questions that are bound to arise in the mind of a reader who is familiar with SUSY. Then the way that these new Multiplets arose from the study of the BRST cohomology is described in Section 5. The Chiral Multiplet and the integration of its auxiliary field is in Sections 6 to 8. The Majorana version of the Half-Chiral Multiplet is explained starting with Section 9 to 14. Section 15 dicusses how and why these Exchange Transformations are special. Then in Section 16, we show how to add a mass term to the Half-Chiral Multiplet. One starts by adding a mass term to the Chiral Multiplet, which was already done in Section 6 . Then the Exchange Transformation is used to convert that Action to the Half-Chiral Multiplet Action. The result is interesting, because it is easy to see how the SUSY Charge is modified, as is explained in Section 18. Interactions in the Half-Chiral Multiplet are discussed in Section 19. Then we introduce the Un-Chiral Multiplet in Section 20 . Again we introduce a mass term by starting with the massive Chiral Multiplet, and ending with the massive Un-Chiral Multiplet in Section 21. This case is simpler than the Half-Chiral Multiplet case, because there are only fermions left in the Un-Chiral Multiplet. Next the well known derivation of the Master Equation for the Chiral Multiplet is recalled in Section 22. This is followed by the parallel derivation for the Un-Chiral Multiplet in Section 23. This latter derivation depends crucially on the invariance of the Master Equation under the Exchange Transformation, as shown in Section 24. Some remarks about the Hamiltonian and the Cohomology are made in Section 25. The Conclusion summarizes the results in section 26. This is followed by a summary of the Dirac version of the Half-Chiral Multiplet in the Appendix, which occupies Sections 27 to 37.

4. Surely the Chiral Multiplet is irreducible, so how can it be reduced? And surely the SUSY algebra implies Degenerate Super-Multiplets [36], so how can there be field theories which do not have them unless SUSY itself is spontaneously or explicitly broken? And if the SUSY charge is not conserved, how can there be any SUSY left at all?

Answer:

- To see in detail how the theory gets around these very reasonable objections, the reader (if he or she is impatient) can look at Section 21 for the Un-Chiral Multiplet case, which is rather simple. The Half-Chiral Multiplet case, which is a little more complicated, is explained in Sections 16, 17 and 18.
- The result of the Exchange Transformation is not a new representation of SUSY. It is a representation of the SUSY algebra in terms of Fields and Zinn Sources, which is physically not the same at all as a representation in terms of Fields alone. Fields are physically realizable, and Zinn Sources are not. When the SUSY algebra spreads to the Zinn Sources, it is no longer realized in the same way on the physical states.
- If Half-Chiral Multiplets or Un-Chiral Multiplets are present, then an attempt to construct a Noether SUSY Current yields an expression which is dependent on the Zinn Sources. So the SUSY Charge cannot be constructed⁴, because the Zinn Sources do not satisfy equations of motion. However, as we shall see, this ‘Suppressed SUSY Charge theory’ still has Bose/Fermi mass degeneracy in the Half-Chiral Multiplet case. But it cannot do so for the Un-Chiral Multiplet, because in that case there are no bosons left in the multiplet.
- The relation of this ‘Suppressed SUSY Charge’ mechanism to the BRST cohomology is rather obscure. This is true even though the examination of BRST cohomology is how it was found, as will be explained below in Section 5. One could say that the SUSY chiral algebra is still present, although the conserved

⁴We recall the Noether analysis: an invariance of the Action leads to a current \mathcal{J}_μ that has zero divergence $\partial^\mu \mathcal{J}_\mu = 0$. Then the integral $Q = \int d^3x \mathcal{J}_0$ satisfies $\frac{d}{dt}Q = 0$, assuming that the surface integral $\int dS^i \mathcal{J}_i$ goes to zero at spatial infinity. The demonstration of this requires use of the Equations of Motion of the Fields. This is explained in most texts on Quantum Field theory.

SUSY charge is not. This puzzling topic, and the question of the Hamiltonian, are discussed a little more in Section 25.

5. BRST Cohomology of the Chiral Multiplet: The BRST cohomology of the Chiral Multiplet is immense, but much of it has unsaturated Lorentz Spinor indices [18]. By and large, this has been taken to mean that ‘*this cohomology cannot appear in a Lorentz invariant Action and so it is of no interest*’. **But this pessimistic view is not valid.** The cohomology is very important, because it can only be relevant if one couples the unsaturated indices to something new, so that the cohomology can appear in an Action. *The point is that this opens up a new view on SUSY.* The simplest such new object is evidently a Chiral Dotted Spinor Superfield[15]. But then the question is whether such an object makes sense by itself, and the answer has been discouraging for a long time. The most obvious Action for the Chiral Dotted Spinor Superfield has higher derivatives and also tachyons. But recently some progress was made on this problem [15,16,17]. A tachyon free Action for the Irreducible Chiral Dotted Spinor Superfield was found and used in a free theory⁵. These Exchange Transformations were found while trying to make that theory an interacting one. In fact they go beyond the results from the cohomology. We will return to the Majorana version of the Irreducible Chiral Dotted Spinor Superfield in Section 9. The Dirac version of the Irreducible Chiral Dotted Spinor Superfield is in the Appendix starting with Section 27.

6. The Chiral Scalar Superfield: We start with the well-known Chiral Multiplet theory. It has the total Action:

$$\mathcal{A}_{\text{Chiral Total}} = \mathcal{A}_{\text{Chiral Kinetic}} + \mathcal{A}_{\text{Chiral Mass and Interaction}} + \mathcal{A}_{\text{Chiral Zinn}} \quad (1)$$

where the free massless kinetic Action is:

$$\mathcal{A}_{\text{Chiral Kinetic}} = \int d^4x \left\{ F\bar{F} - \psi_\alpha \partial^{\alpha\dot{\beta}} \bar{\psi}_{\dot{\beta}} + \frac{1}{2} \partial_{\alpha\dot{\beta}} A \partial^{\alpha\dot{\beta}} \bar{A} \right\} \quad (2)$$

and mass and interaction terms look like⁶

$$\mathcal{A}_{\text{Chiral Mass and Interaction}} = \int d^4x \left\{ m_1 A F - \frac{1}{2} m_1 \psi^\alpha \psi_\alpha + g_1 A^2 F - g_1 A \psi^\alpha \psi_\alpha + * \right\} \quad (3)$$

and the Zinn Action is:

$$\mathcal{A}_{\text{Chiral Zinn}} = \left\{ \Gamma (C^\alpha \psi_\alpha + \xi \cdot \partial A) + Y^\alpha \left(\bar{C}^{\dot{\delta}} \partial_{\alpha\dot{\delta}} A + C_\alpha F + \xi \cdot \partial \psi_\alpha \right) \right. \quad (4)$$

$$\left. + \Lambda \left(\bar{C}^{\dot{\beta}} \partial_{\alpha\dot{\beta}} \psi^\alpha + \xi \cdot \partial F \right) + * \right\} + \mathcal{A}_{\text{SUSY}} \quad (5)$$

where

$$\mathcal{A}_{\text{SUSY}} = -C^\alpha \bar{C}^{\dot{\alpha}} h_{\alpha\dot{\alpha}} \quad (6)$$

Here A is a complex Scalar Field, ψ_α is a two-component Weyl Spinor Field, and F is a complex auxiliary Scalar Field. The Zinn Action has the form $\int d^4x \{ \Gamma \delta A + Y \delta \psi + \Lambda \delta F + * \}$, which is a sum of Zinn Sources coupled to the SUSY variations of the Fields.

The SUSY invariance can be summarized by the fact that the above Action $\mathcal{A}_{\text{Total}}$ yields zero for the Master Equation:

$$\mathcal{P}_{\text{Chiral}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta A} \frac{\delta \mathcal{A}}{\delta \Gamma} + \frac{\delta \mathcal{A}}{\delta \psi_\alpha} \frac{\delta \mathcal{A}}{\delta Y^\alpha} + \frac{\delta \mathcal{A}}{\delta F} \frac{\delta \mathcal{A}}{\delta \Lambda} + * \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (7)$$

⁵That progress was not very exciting however, because it dealt only with free theories. Moreover, efforts to make those theories interacting have been fraught with obstructions of the kind mentioned in [16]. However this changes if one integrates the auxiliary W , as will be seen below.

⁶Here $+$ means ‘add the Complex Conjugate of the previous terms’.

We define

$$\xi \cdot \partial \equiv \xi^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \quad (8)$$

The following terms⁷ must always be added to every Master Equation of SUSY theories:

$$\mathcal{P}_{\text{SUSY}}[\mathcal{A}] = \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} = -C_\alpha \bar{C}_{\dot{\alpha}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (9)$$

These terms take account of the fact that the SUSY algebra closes onto a translation. The BRST ghost of that translation for rigid SUSY is the constant anticommuting vector $\xi^{\alpha\dot{\alpha}}$. Equation (9) summarizes the fact that two SUSY transformations with the SUSY parameter C_α act like a spacetime translation.

7. The Chiral Scalar Theory after Integration of the Auxiliary F After integrating the F auxiliary Field in Section 6, and dropping the Source Λ for its variation, one gets⁸.

$$\mathcal{A}_{\text{Chiral F Int}} = \int d^4x \left\{ -\psi_\alpha \partial^{\alpha\dot{\beta}} \bar{\psi}_{\dot{\beta}} + \frac{1}{2} \partial_{\alpha\dot{\beta}} A \partial^{\alpha\dot{\beta}} \bar{A} + \Gamma (C^\alpha \psi_\alpha + \xi \cdot \partial A) \right. \quad (10)$$

$$\left. + Y^\alpha (\bar{C}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} A + \xi \cdot \partial \psi_\alpha) + \bar{\Gamma} (\bar{C}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} + \xi \cdot \partial \bar{A}) + \bar{Y}^{\dot{\alpha}} (C^\alpha \partial_{\alpha\dot{\alpha}} \bar{A} + \xi \cdot \partial \bar{\psi}_{\dot{\alpha}}) \right. \quad (11)$$

$$\left. - \frac{1}{2} m_1 \psi^\alpha \psi_\alpha - g_1 A \psi^\alpha \psi_\alpha - \frac{1}{2} \bar{m}_1 \bar{\psi}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} - \bar{g}_1 \bar{A} \bar{\psi}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} \right. \quad (12)$$

$$\left. - (\bar{m}_1 \bar{A} + \bar{g}_1 \bar{A}^2 + \bar{Y}^{\dot{\alpha}} \bar{C}_{\dot{\alpha}}) (m_1 A + g_1 A^2 + Y^\alpha C_\alpha) \right\} \quad (13)$$

which yields zero for the smaller Master Equation:

$$\mathcal{P}_{\text{Chiral F Int}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta \bar{A}} \frac{\delta \mathcal{A}}{\delta \bar{\Gamma}} + \frac{\delta \mathcal{A}}{\delta \bar{\psi}_{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \bar{Y}^{\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta A} \frac{\delta \mathcal{A}}{\delta \Gamma} + \frac{\delta \mathcal{A}}{\delta \psi_\alpha} \frac{\delta \mathcal{A}}{\delta Y^\alpha} \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (14)$$

This is the Master Equation for a Chiral Multiplet where the auxiliary F has been integrated out. We will see this form again below. The Zinn Sources Y appear quadratically, and they keep the invariance intact. Now we shall put this derivation in a theorem that we will often need:

8. Theorem about Auxiliary Fields and Master Equations: The technique in the above example is frequently used in this paper, and it is worth making it into a theorem. We will use this theorem repeatedly in this paper and the paper [34].

Theorem 1: Given an Action \mathcal{A} that satisfies a given Master Equation \mathcal{P} , then

1. Suppose that there is a Field F in that Action⁹ which has an algebraically invertible quadratic term, and a linear term¹⁰ in F , so that the total Action has the form:

$$\mathcal{A} = \int d^4x \left\{ m_{ij} F^i F^j + F^i G_i + \Lambda_i \delta F^i + \text{etc.} \right\} + \mathcal{A}_{\text{Other Terms}} \quad (15)$$

and that the Master Equation has the form

$$\mathcal{P}[\mathcal{A}] = \int d^4x \left(\frac{\delta \mathcal{A}}{\delta \Lambda_i} \frac{\delta \mathcal{A}}{\delta F^i} \right) + \mathcal{P}_{\text{Other Terms}}[\mathcal{A}] \quad (16)$$

⁷ One cannot implement the BRST method unless one closes the algebra. But there is a genuine problem here. Although there are identities which arise from this Master Equation that are true in perturbation theory for this rigid SUSY theory, those that involve ξ are not true, because ξ is not integrated in the Feynman path integral, although it is more like a Field than a Zinn Source. This problem can be resolved by embedding the rigid theory in Supergravity.

⁸ Insert the Action into a Feynman path integral with Sources for the Fields, as in Section 22 below, and derive the Master Equation in the usual way. Then complete the quadratic in F and \bar{F} and perform the same exercise, after dropping the Source Λ . Then shift the F and integrate it, which just leaves a number. This leaves the terms and Zinn Action shown.

⁹ Here we assume that F is real. In Section 7, F was complex.

¹⁰ Auxiliary Fields generally satisfy this condition.

2. Then we can integrate out the Field F and get a new Action and a new Master Equation as follows:

- (a) Remove the Zinn Source term $\int d^4x \Lambda_i \delta F^i$ from the Action (15), and
- (b) Remove the related term $\frac{\delta \mathcal{A}}{\delta \Lambda_i} \frac{\delta \mathcal{A}}{\delta F^i}$ from the Master Equation (16),
- (c) The new Action is

$$\mathcal{A}_{\text{New}} = \int d^4x \frac{-1}{4} \{ (m^{-1})^{ij} G_i G_j \} + \mathcal{A}_{\text{Other Terms}} \quad (17)$$

- (d) The new Action yields zero for the new Master Equation, which reduces to

$$\mathcal{P}_{\text{New}}[\mathcal{A}] = \mathcal{P}_{\text{Other Terms}}[\mathcal{A}] \quad (18)$$

The proof is simple. Write the relevant terms in the Action (15) in the form

$$\mathcal{A} = \int d^4x \left\{ m_{ij} \left(F^i + \left(\frac{m^{-1}}{2} G \right)^i \right) \left(F^j + \left(\frac{m^{-1}}{2} G \right)^j \right) - m_{ij} \left(\frac{m^{-1}}{2} G \right)^i \left(\frac{m^{-1}}{2} G \right)^j \right\} \quad (19)$$

Then shift and integrate the Field F^i , after placing it in a Feynman path integral as in Sections 22 and 23 below. This yields an irrelevant constant plus the second term. It is important that the G^i does not contain F^i . But it can contain anything else including Zinn Sources.

The usual demonstration that the Master Equation yields zero, as in Sections 22 and 23 below, goes through when this has been done, and no Zinn Source for the variation of F^i is needed because F^i is gone from the theory. This theorem is important because our Exchange Transformations here map Actions which have had their auxiliaries integrated, as we shall see.

9. The Majorana Irreducible Chiral Dotted Spinor Supermultiplet: Next we consider the simplest example of a Majorana Irreducible Chiral Dotted Spinor Supermultiplet. It is the Majorana version of the Dirac type theory¹¹ that was discussed in [15,16,17]. We will use the notation of [15]. The Action is quite simple:

$$\mathcal{A}_{\text{MI}} = \mathcal{A}_{\text{MI Kinetic}} + \mathcal{A}_{\text{MI Zinn}} \quad (20)$$

where

$$\mathcal{A}_{\text{MI Kinetic}} = \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} - \frac{1}{2} W_{\alpha\dot{\alpha}} W^{\alpha\dot{\alpha}} + \frac{1}{2} G \square G - \frac{1}{\sqrt{2}} \eta \left(\phi^{\dot{\delta}} \bar{C}_{\dot{\delta}} + \bar{\phi}^{\dot{\delta}} C_{\dot{\delta}} \right) \right\} \quad (21)$$

In the above, G is a real Scalar Field, $\phi^{\dot{\alpha}}$ is a two component complex Weyl Spinor, W_{μ} is a real vector Field¹², and it turns out to be auxiliary (no kinetic term), η is a real Grassmann odd Antighost Scalar Field and C_{α} is again the Grassmann even space-time constant Weyl Spinor ghost Field. The Zinn Action that we need is:

$$\begin{aligned} \mathcal{A}_{\text{MI Zinn}} = & \int d^4x Z^{\dot{\alpha}} \left(-i \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} G C^{\alpha} - W_{\alpha\dot{\alpha}} C^{\alpha} + \xi \cdot \partial \phi_{\dot{\alpha}} \right) + \bar{Z}^{\alpha} \left(i \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} G \bar{C}^{\dot{\alpha}} - W_{\alpha\dot{\alpha}} \bar{C}^{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}_{\alpha} \right) \\ & + \Sigma^{\alpha\dot{\alpha}} \left(\sqrt{2} \eta \bar{C}_{\dot{\alpha}} C_{\alpha} - \frac{1}{2} \partial_{\alpha}^{\dot{\gamma}} \phi_{\dot{\gamma}} \bar{C}_{\dot{\alpha}} - \frac{1}{2} \partial_{\alpha}^{\dot{\gamma}} \phi_{\dot{\alpha}} \bar{C}_{\dot{\gamma}} - \frac{1}{2} \partial_{\alpha}^{\gamma} \bar{\phi}_{\gamma} C_{\alpha} - \frac{1}{2} \partial_{\alpha}^{\gamma} \bar{\phi}_{\alpha} C_{\gamma} + \xi \cdot \partial W_{\alpha\dot{\alpha}} \right) \\ & + \Upsilon \left(-\frac{i}{\sqrt{2}} \bar{\phi}_{\beta} C^{\beta} + \frac{i}{\sqrt{2}} \phi_{\dot{\beta}} \bar{C}^{\dot{\beta}} + \xi \cdot \partial G \right) + J \left(\frac{1}{\sqrt{2}} \partial_{\gamma\dot{\delta}} W^{\gamma\dot{\delta}} + \xi \cdot \partial \eta \right) + \mathcal{A}_{\text{SUSY}} \end{aligned} \quad (22)$$

¹¹This could be derived in exactly the same way as in [15,16,17], using BRST recycling, starting with the U(1) Gauge theory in this Majorana case. We start with this simplest case but the other case is also needed and we return to it below in Section 27

¹² $W_{\alpha\dot{\alpha}} = W_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu}$

Any of the three Actions in (20) yields zero when inserted into the following Master Equation for SUSY:

$$\mathcal{P}_{\text{MI}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z^{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \bar{Z}^{\alpha}} \frac{\delta \mathcal{A}}{\delta \bar{\phi}_{\alpha}} + \frac{\delta \mathcal{A}}{\delta \Sigma^{\alpha\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta W_{\alpha\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \Upsilon} \frac{\delta \mathcal{A}}{\delta G} + \frac{\delta \mathcal{A}}{\delta J} \frac{\delta \mathcal{A}}{\delta \eta} \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (23)$$

10. Integrate the auxiliary out of the Irreducible Chiral Dotted Spinor Supermultiplet: After [15,16,17] were written, the main problem for the new version of the Irreducible Chiral Dotted Spinor Superfield was whether it could be put into an interacting Action. That seemed very difficult at first. However, it turns out that interactions can be generated easily if we first integrate the auxiliary vector Field $W_{\alpha\dot{\beta}}$ out of the Action for the Majorana Irreducible Chiral Dotted Spinor Superfield in Section 9. We will give the result of that a new name: the Half-Chiral Multiplet Action. We use that name for the Half-Chiral Multiplet because it has half the Scalar degrees of freedom that a Chiral Multiplet has.

Once the integration of W is done, it is fairly easy to recognize that the resulting Half-Chiral Multiplet Action is really the result of an Exchange Transformation acting on a Chiral Multiplet that has had its auxiliary Field F integrated. This is the Exchange Transformation that we will be using. Because it will turn out that this theory can be obtained from an Exchange Transformation acting on a Chiral Multiplet, we can couple the Chiral Multiplet using known methods, and then using the inverse Exchange Transformation, we can deduce the interactions of the Half-Chiral Multiplet.

11. The Majorana Half-Chiral Multiplet: Drop the Σ terms in the action in Equation (20) in Section 9, and integrate W out of the Action¹³. This yields a closely related Action, which we will dignify by a new name, the Majorana Half-Chiral Multiplet:

$$\begin{aligned} \mathcal{A}_{\text{MHC}} = & \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + \frac{1}{2} G \square G - \frac{1}{\sqrt{2}} \eta \left(\phi^{\dot{\delta}} \bar{C}_{\dot{\delta}} + \bar{\phi}^{\delta} C_{\delta} \right) \right\} \\ & + \int d^4x \left\{ Z^{\dot{\alpha}} \left(-i \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} G C^{\alpha} + \xi \cdot \partial \phi_{\dot{\alpha}} \right) + \bar{Z}^{\alpha} \left(i \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} G \bar{C}^{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}_{\alpha} \right) \right. \\ & \quad \left. + \Upsilon \left(-\frac{i}{\sqrt{2}} \bar{\phi}_{\beta} C^{\beta} + \frac{i}{\sqrt{2}} \phi_{\dot{\beta}} \bar{C}^{\dot{\beta}} + \xi \cdot \partial G \right) + J \xi \cdot \partial \eta \right\} \\ & + \frac{1}{2} \int d^4x \left\{ Z^{\dot{\alpha}} C^{\alpha} + \bar{Z}^{\alpha} \bar{C}^{\dot{\alpha}} + \frac{1}{\sqrt{2}} \partial^{\alpha\dot{\alpha}} J \right\} \left\{ Z_{\dot{\alpha}} C_{\alpha} + \bar{Z}_{\alpha} \bar{C}_{\dot{\alpha}} + \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} J \right\} + \mathcal{A}_{\text{SUSY}} \end{aligned} \quad (24)$$

which we can rearrange to

$$\begin{aligned} \mathcal{A}_{\text{MHC}} = & \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + \frac{1}{2} G \square G + \frac{1}{2} J \square J - \frac{1}{\sqrt{2}} \eta \left(\phi^{\dot{\delta}} \bar{C}_{\dot{\delta}} + \bar{\phi}^{\delta} C_{\delta} \right) \right. \\ & \quad \left. + \Upsilon \left(-\frac{i}{\sqrt{2}} \bar{\phi}_{\beta} C^{\beta} + \frac{i}{\sqrt{2}} \phi_{\dot{\beta}} \bar{C}^{\dot{\beta}} + \xi \cdot \partial G \right) \right\} \\ & + \int d^4x Z^{\dot{\alpha}} \left(C^{\alpha} \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} J - i \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} G C^{\alpha} + \xi \cdot \partial \phi_{\dot{\alpha}} \right) \\ & + \bar{Z}^{\alpha} \left(\bar{C}^{\dot{\alpha}} \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} J + i \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} G \bar{C}^{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}_{\alpha} \right) \\ & + \int d^4x \left\{ Z^{\dot{\alpha}} C^{\alpha} \bar{Z}_{\alpha} \bar{C}_{\dot{\alpha}} + J \xi \cdot \partial \eta \right\} + \mathcal{A}_{\text{SUSY}} \end{aligned} \quad (25)$$

¹³Insert the Action into a Feynman path integral with Sources for the Fields, and derive the Master Equation in the usual way. Then complete the quadratic in W and perform the same exercise, while leaving its Source out, and shifting the W to integrate it. This leaves the terms and Zinn Action shown. This is an application of the theorem in Section 8.

Now this yields zero for:

$$\mathcal{P}_{\text{MHC}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z^{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \bar{Z}^{\alpha}} \frac{\delta \mathcal{A}}{\delta \bar{\phi}_{\alpha}} + \frac{\delta \mathcal{A}}{\delta \Upsilon} \frac{\delta \mathcal{A}}{\delta G} + \frac{\delta \mathcal{A}}{\delta J} \frac{\delta \mathcal{A}}{\delta \eta} \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (26)$$

12. Remarkable Symmetries of the Action \mathcal{A}_{MHC} : The above Action \mathcal{A}_{MHC} in Equation (25) has a remarkable symmetry which was not obvious before we integrated the auxiliary W . The Field η and the Source Υ appear in similar ways. The term $\eta \left(\frac{1}{\sqrt{2}} \bar{\phi}_{\delta} C^{\delta} + \frac{1}{\sqrt{2}} \phi_{\delta} \bar{C}^{\delta} \right)$ looks like a Zinn Source coupled to a variation, except that η is a Field. The Field G and the Source J also appear in similar ways. The term $\frac{1}{2} J \square J$ looks like a kinetic term for J , except that J is a Source.

13. New Variables for the Majorana Half-Chiral Multiplet This symmetry can be exploited with a Generating Functional for an Exchange Transformation of the Action and Master Equation. The new Action will yield zero for the new Master Equation. Our new Action will have ‘a new complex Field’ (S, \bar{S}) and ‘a new complex Zinn Source’ $(\Gamma, \bar{\Gamma})$. These will replace the ‘old real Fields’ (η, G) and the ‘old real Zinn Sources’ (J, Υ) . We will choose a Generating Functional of the new Zinn Sources $(\Gamma, \bar{\Gamma})$ and the old Field G and the old Zinn Source J .

$$\mathcal{G}_{\text{MHC}} = \int d^4x \left\{ \frac{1}{\sqrt{2}} \Gamma (J - iG) + \frac{1}{\sqrt{2}} \bar{\Gamma} (J + iG) \right\} \quad (27)$$

and the Exchange Transformations are:

$$S = \frac{\delta \mathcal{G}}{\delta \Gamma} = \frac{1}{\sqrt{2}} (J - iG); \quad \bar{S} = \frac{\delta \mathcal{G}}{\delta \bar{\Gamma}} = \frac{1}{\sqrt{2}} (J + iG); \quad (28)$$

$$\eta = \frac{\delta \mathcal{G}}{\delta J} = \frac{1}{\sqrt{2}} (\Gamma + \bar{\Gamma}); \quad \Upsilon = \frac{\delta \mathcal{G}}{\delta G} = \frac{1}{\sqrt{2}} (-i\Gamma + i\bar{\Gamma}) \quad (29)$$

These have the following inverses:

$$G = \frac{1}{\sqrt{2}} (iS - i\bar{S}); \quad J = \frac{1}{\sqrt{2}} (S + \bar{S}); \quad \Gamma = \frac{1}{\sqrt{2}} (\eta + i\Upsilon); \quad \bar{\Gamma} = \frac{1}{\sqrt{2}} (\eta - i\Upsilon) \quad (30)$$

14. New Action after Exchange Transformation: It looks like the Chiral Action The new Action expressed in terms of the new variables is:

$$\begin{aligned} \mathcal{A}_{\text{CM}} = \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + S \square \bar{S} + \bar{\Gamma} (\bar{\phi}_{\delta} C^{\delta} + \xi \cdot \partial \bar{S}) + \Gamma (\phi_{\delta} \bar{C}^{\delta} + \xi \cdot \partial S) \right. \\ \left. + Z^{\dot{\alpha}} (\partial_{\alpha\dot{\alpha}} S C^{\alpha} + \xi \cdot \partial \phi_{\dot{\alpha}}) + \bar{Z}^{\alpha} (\partial_{\alpha\dot{\alpha}} \bar{S} \bar{C}^{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}_{\alpha}) - Z_{\dot{\alpha}} C^{\alpha} \bar{Z}_{\alpha} \bar{C}^{\dot{\alpha}} \right\} \end{aligned} \quad (31)$$

The new Master Equation is:

$$\mathcal{P}_{\text{CM}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z^{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \Gamma} \frac{\delta \mathcal{A}}{\delta S} + * \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (32)$$

The invariance of the new Action \mathcal{A}_{CM} is expressed by:

$$\mathcal{P}_{\text{CM}}[\mathcal{A}_{\text{CM}}] = 0 \quad (33)$$

Note that the expressions in this Section are identical to the results for the Chiral Multiplet in Section 7 above, if one changes the names of the Fields and Zinn Sources, and sets $m_1 = g_1 = 0$ in Section 7. Here is the mapping from Section 7 to this Section 14.

$$A \rightarrow \bar{S}; \quad \Gamma \rightarrow \bar{\Gamma}; \quad Y^{\alpha} \rightarrow \bar{Z}^{\alpha}; \quad \psi_{\alpha} \rightarrow \bar{\phi}_{\alpha} \quad (34)$$

and their Complex Conjugates.

15. Poisson Brackets, Canonical Transformations, Exchange Transformations, and the Master Equation. The Master Equation [26,27,28,29,30,31] has the same form as a Poisson Bracket¹⁴ in classical mechanics [37,38]. There is no essential distinction between coordinates and momentum for classical mechanics, but there is one for the Master Equation. The reason is that the Fields are quantized and the Zinn Sources are not. Nevertheless, Canonical Transformations play a role for both kinds of Poisson Brackets, because they leave the Poisson Bracket invariant [37,38]. For the Master Equation case we are calling these Canonical Transformations ‘Exchange Transformations’, because they can map one action to another, which a Canonical Transformation would never do in Classical Mechanics. But it is important to remember that these Exchange Transformations must always yield an Action which yields zero for the resulting Master Equation, and that is because they are Canonical Transformations in their mathematical form.

16. Finding Mass and Interaction Terms for the Half-Chiral Multiplet by Starting with the Known Mass and Interaction Terms for the Chiral Multiplet: So now we see that the Half-Chiral Multiplet arises from the Chiral Multiplet through the Exchange Transformation above. This is useful because we know how to make masses and interactions for the Chiral Multiplet, and we did this in Section 7. Can we put those into a Chiral Multiplet and then use the Exchange Transformation to deduce what they look like for the Half-Chiral Multiplet from that? The answer is yes! Let us see how this works in detail, by adding a mass term and a cubic interaction term to (31). This is a little tricky, because the two theories are related by an Exchange Transformation only when they have both had their auxiliaries integrated, and the auxiliaries are different—the Chiral Multiplet has a Scalar F and the Irreducible Chiral Dotted Spinor Superfield has a vector auxiliary $W_{\alpha\dot{\alpha}}$. When these auxiliaries are integrated then there is a Exchange Transformation that relates the two, and we call the Irreducible Chiral Dotted Spinor Superfield with its W auxiliary integrated, by the shorter and more descriptive name Half-Chiral Multiplet. The Chiral Multiplet with a mass term and an interaction term has the following Action, once the auxiliary has been integrated:

$$\begin{aligned} \mathcal{A}_{\text{CM with Mass \& Interaction}} = \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + S \square \bar{S} + \bar{\Gamma} (\bar{\phi}_{\dot{\delta}} C^{\dot{\delta}} + \xi \cdot \partial \bar{S}) \right. \\ \left. + \Gamma (\phi_{\dot{\delta}} \bar{C}^{\dot{\delta}} + \xi \cdot \partial S) + Z^{\dot{\alpha}} (\partial_{\alpha\dot{\alpha}} S C^{\alpha} + \xi \cdot \partial \phi_{\dot{\alpha}}) + \bar{Z}^{\alpha} (\partial_{\alpha\dot{\alpha}} \bar{S} \bar{C}^{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}_{\alpha}) \right. \\ \left. - \left(\frac{1}{2} m_1 + g_1 S \right) \phi^{\dot{\alpha}} \phi_{\dot{\alpha}} - \left(\frac{1}{2} \bar{m}_1 + \bar{g}_1 \bar{S} \right) \bar{\phi}^{\alpha} \bar{\phi}_{\alpha} - \left(m_1 S + g_1 S^2 + Z_{\dot{\alpha}} \bar{C}^{\dot{\alpha}} \right) \left(m_1 \bar{S} + \bar{g}_1 \bar{S}^2 + \bar{Z}_{\alpha} C^{\alpha} \right) \right\} \quad (35) \end{aligned}$$

We are using the notation in Section 14, rather than the notation in Sections 6 and 7. This is done to agree with the notation in [34].

17. The Half-Chiral Multiplet Action with Mass and Interaction It is elementary to use the Exchange Transformation from Section 13 on the expression in Section 16. The result is the Action for the massive interacting Majorana Half-Chiral Multiplet:

$$\begin{aligned} \mathcal{A}_{\text{MHC with Mass \& Interaction}} = \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + \frac{1}{\sqrt{2}} (J - iG) \square \frac{1}{\sqrt{2}} (J + iG) \right. \\ \left. + \frac{1}{\sqrt{2}} (\eta - i\Upsilon) \left(\bar{\phi}_{\dot{\delta}} C^{\dot{\delta}} + \xi \cdot \partial \frac{1}{\sqrt{2}} (J + iG) \right) + \frac{1}{\sqrt{2}} (\eta + i\Upsilon) \left(\phi_{\dot{\delta}} \bar{C}^{\dot{\delta}} + \xi \cdot \partial \frac{1}{\sqrt{2}} (J - iG) \right) \right. \\ \left. + Z_{\dot{\alpha}} \left(\partial^{\alpha\dot{\alpha}} \frac{1}{\sqrt{2}} (J - iG) C_{\alpha} + \xi \cdot \partial \phi^{\dot{\alpha}} \right) + \bar{Z}_{\alpha} \left(\partial^{\alpha\dot{\alpha}} \frac{1}{\sqrt{2}} (J + iG) \bar{C}_{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}^{\alpha} \right) \right\} \end{aligned}$$

¹⁴The Master Equation is a Poisson Bracket, except that it uses Grassmann anticommuting quantities (these actually simplify things somewhat).

$$-\left(\frac{1}{2}m_1 + g_1\frac{1}{\sqrt{2}}(J - iG)\right)\phi^{\dot{\alpha}}\phi_{\dot{\alpha}} - \left(\frac{1}{2}\bar{m}_1 + \bar{g}_1\frac{1}{\sqrt{2}}(J + iG)\right)\bar{\phi}^{\alpha}\bar{\phi}_{\alpha} \quad (36)$$

$$- \left(m_1\frac{1}{\sqrt{2}}(J - iG) + g_1\frac{1}{2}(J - iG)^2 + Z_{\dot{\alpha}}\bar{C}^{\dot{\alpha}} \right) \left(m_1\frac{1}{\sqrt{2}}(J + iG) + \bar{g}_1\frac{1}{2}(J + iG)^2 + \bar{Z}_{\alpha}C^{\alpha} \right) \quad (37)$$

18. Details for the Majorana Half-Chiral Multiplet with mass, and the Loss of the SUSY Charge Let us set $g_1 \rightarrow 0$ in Section 17. Then the expression in Section 17 is the Half-Chiral Multiplet with just a Majorana mass term. The only difference from the massless case is:

$$\mathcal{A}_{\text{MHC with Mass}} = \int d^4x \left\{ \dots \right. \quad (38)$$

$$\left. -\frac{1}{2}m_1\phi^{\dot{\alpha}}\phi_{\dot{\alpha}} - \frac{1}{2}m_1\bar{\phi}^{\alpha}\bar{\phi}_{\alpha} - \left(m_1\frac{1}{\sqrt{2}}(J - iG) + Z_{\dot{\alpha}}C^{\dot{\alpha}} \right) \left(m_1\frac{1}{\sqrt{2}}(J + iG) + \bar{Z}_{\alpha}\bar{C}^{\alpha} \right) \right\} \quad (39)$$

We see that indeed there is a mass term here for the Scalar, namely

$$\mathcal{A}_{\text{MHC with Mass}} = \int d^4x \left\{ \dots - \frac{m_1^2}{2}(J^2 + G^2) \right\} \quad (40)$$

But note that there is also a ‘mass’ term for the Zinn Source J in Equation (40), and then there are extra terms that are all in the Zinn Action

$$\mathcal{A}_{\text{MHC with Mass}} = \int d^4x \left\{ \dots - (Z_{\dot{\alpha}}C^{\dot{\alpha}}) \left(\bar{Z}_{\alpha}\bar{C}^{\alpha} \right) \right. \quad (41)$$

$$\left. - \left(\frac{m_1}{\sqrt{2}}(J - iG) \right) \left(\bar{Z}_{\alpha}\bar{C}^{\alpha} \right) - (Z_{\dot{\alpha}}C^{\dot{\alpha}}) \left(\frac{m_1}{\sqrt{2}}(J + iG) \right) \right\} \quad (42)$$

So here is what we have discovered: The Half-Chiral Multiplet Action has one Scalar G and a Majorana Spinor ϕ and also the Source J . When we generate the Half-Chiral Multiplet mass term from the massive Chiral Multiplet plus the Exchange Transformation, we get a massive Spinor and a massive Scalar G . We also get an object that looks like a mass term for J , but J is a Zinn Source.

Because this is all a result of the Exchange Transformation, we are guaranteed that it will satisfy the Half-Chiral Master Equation in Equation (26).

But just looking at it we can see that it describes a Multiplet of SUSY that has one Scalar and a Spinor—but this is clearly not a proper mass multiplet that forms a representation of the SUSY algebra—that needs two Scalars and there is only one here. And yet there is some mass degeneracy here, as though the SUSY algebra is ‘half-present’.

And that is the essential point of this entire paper! The Exchange Transformation has enabled us to build a SUSY theory that does not have a nice conserved Noether Charge—the physical theory here is not the proper one we would expect from a theory with a conserved Noether current—it has this Source J where the Scalar should be. And because of the Exchange Transformation, it has the right set of Zinn Source terms to satisfy the Half-Chiral Master Equation.

19. Details for the Half-Chiral Multiplet with Mass and Interactions Now consider the case where $g_1 \neq 0$ in Section 17. The Action there has both mass and interactions. Note the complicated way that the Zinn Source is intertwined with the Scalar Field. We could do the same with a Chiral Action that also includes any other kind of interactions, with Gauge, other Chiral Multiplets, even Supergravity. We would end up with a lot of Zinn Source terms and an Action that only goes half-way towards a representation of the SUSY algebra.

The Action in Section 17 is probably the simplest possible Half-Chiral massive interacting theory, and it would be worth while to examine its nilpotent BRST operator δ_{BRST} (this is the ‘square root’ of the Master Equation), and its one loop diagrams to get a feel for how this Half-Chiral Multiplet works at one loop.

These Half-Chiral Multiplets will be used in [34] for the Higgs Multiplets. We will use a Dirac Half-Chiral Multiplet and a Majorana Half-Chiral Multiplet there.

20. Un-Chiral Multiplets If we take the Exchange Transformation that goes all the way, to generate an Un-Chiral Multiplet, we get a theory with no Scalar and some interesting Zinn terms, and a mass just for the Spinor. That case is actually simpler than the above, and we write it down in Section 21. In that case the interaction term would just add to the Zinn Source sector, and the theory would still be a free massive theory. To get interactions there requires Gauge theory.

The Exchange Transformation can be applied twice, so that both Scalars are removed from the Lagrangian. The new Lagrangian gains two terms with Antighosts while the Fermions remain as quantized Fields. Start with the Chiral Multiplet with the Action in Equation (31). Now consider using an Exchange Transformation generated as follows. Instead of the Exchange Transformation in Section 13 we now note that the old ‘Fields’ (S, \bar{S}) are conjugate to the old ‘Zinn Sources’ $(\Gamma, \bar{\Gamma})$, and we want an Exchange Transformation that takes us to the new ‘Fields’ $(\eta, \bar{\eta})$ which are conjugate to the new ‘Zinn Sources’ (J, \bar{J}) . We choose a generating functional of the new Zinn Sources (J, \bar{J}) and the old ‘Zinn Sources’ $(\Gamma, \bar{\Gamma})$. This is

$$\mathcal{G}_{\text{MUC}} = \int d^4x \{ \Gamma J + \bar{\Gamma} \bar{J} \} \quad (43)$$

and we have

$$S \rightarrow \frac{\delta \mathcal{G}}{\delta \Gamma} = J; \quad \bar{S} \rightarrow \frac{\delta \mathcal{G}}{\delta \bar{\Gamma}} = \bar{J}; \quad (44)$$

and

$$\Gamma = \frac{\delta \mathcal{G}}{\delta J} \rightarrow \eta; \quad \bar{\Gamma} = \frac{\delta \mathcal{G}}{\delta \bar{J}} \rightarrow \bar{\eta}; \quad (45)$$

We get the following transformed Action:

$$\begin{aligned} \mathcal{A}_{\text{MUC}} = \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + J \square \bar{J} + \bar{\eta} (\bar{\phi}_{\dot{\delta}} C^{\dot{\delta}} + \xi \cdot \partial \bar{J}) + \eta \left(\phi_{\dot{\delta}} \bar{C}^{\dot{\delta}} + \xi \cdot \partial J \right) \right. \\ \left. + Z_{\dot{\alpha}} (\partial^{\alpha\dot{\alpha}} J C_{\alpha} + \xi \cdot \partial \phi^{\dot{\alpha}}) + \bar{Z}_{\alpha} (\partial^{\alpha\dot{\alpha}} \bar{J} \bar{C}_{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}^{\alpha}) - Z_{\alpha} C^{\alpha} \bar{Z}_{\alpha} \bar{C}^{\alpha} \right\} \end{aligned} \quad (46)$$

which yields zero for the new Master Equation

$$\mathcal{P}_{\text{MUC}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z^{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \bar{Z}^{\alpha}} \frac{\delta \mathcal{A}}{\delta \bar{\phi}_{\alpha}} + \frac{\delta \mathcal{A}}{\delta \bar{\eta}} \frac{\delta \mathcal{A}}{\delta \bar{J}} + \frac{\delta \mathcal{A}}{\delta J} \frac{\delta \mathcal{A}}{\delta \eta} \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (47)$$

This looks very similar to (31), but the theory is not at all the same. The J are not quantized and the η are quantized. So the complex quantized Scalar Field (S, \bar{S}) is gone from the theory along with its Zinn Source $(\Gamma, \bar{\Gamma})$, while the quantized Fermion $(\phi, \bar{\phi})$ remains, and the new Zinn Sources (J, \bar{J}) and quantized Antighosts $(\eta, \bar{\eta})$ appear. We are assured that the new Action satisfies the new Master Equation, because the old Action satisfied the old Master Equation.

This procedure does not correspond to any known starting Action like the Half-Chiral Multiplet discussed above in section 11. Equation (46) is a new Action. We started with the Half-Chiral Multiplets found by BRST recycling, and then found the Exchange Transformations that took those theories to Chiral Multiplets. Now by generalizing those Exchange Transformations we have discovered new theories that have no physical Scalars at all, just physical Fermions.

21. Mass Term for the Un-Chiral Multiplet In Section 18 above, we discussed the mass term for the Half-Chiral Multiplet. It has a mass term that only goes half-way towards that of a Chiral Multiplet. Now let us start again with the Chiral Multiplet with the Action in Equation (31), with a mass term, so that we

get Equation (35), and then use the Exchange Transformation in Section 20 on Equation (35). This yields the following Action (we are setting $g_1 \rightarrow 0$):

$$\begin{aligned} \mathcal{A}_{\text{Majorana UnChiral Massive}} = \int d^4x \left\{ -\phi^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}^{\alpha} + J \square \bar{J} + \bar{\eta} (\bar{\phi}_{\dot{\delta}} C^{\dot{\delta}} + \xi \cdot \partial \bar{J}) + \eta \left(\phi_{\dot{\delta}} \bar{C}^{\dot{\delta}} + \xi \cdot \partial J \right) \right. \\ \left. + Z_{\dot{\alpha}} (\partial^{\alpha\dot{\alpha}} J C_{\alpha} + \xi \cdot \partial \phi^{\dot{\alpha}}) + \bar{Z}_{\alpha} (\partial^{\alpha\dot{\alpha}} \bar{J} C_{\dot{\alpha}} + \xi \cdot \partial \bar{\phi}^{\alpha}) \right. \\ \left. - m_1 \frac{1}{2} \phi^{\dot{\alpha}} \phi_{\dot{\alpha}} - m_1 \frac{1}{2} \bar{\phi}^{\alpha} \bar{\phi}_{\alpha} - (m_1 J + Z_{\dot{\alpha}} C^{\dot{\alpha}}) (m_1 \bar{J} + \bar{Z}_{\alpha} \bar{C}^{\alpha}) \right\} \end{aligned} \quad (48)$$

The Scalar Mass terms $S (\square - m_1 \bar{m}_1) \bar{S}$ in Equation (35) have become the Zinn Source term $J (\square - m_1 \bar{m}_1) \bar{J}$ in Equation (48). So Equation (48) is an Action which has a Fermionic Mass term and no Scalars. Note that it is quite a lot simpler than the Half-Chiral Multiplet. We are guaranteed that it will satisfy the appropriate Master Equation for the Un-Chiral Multiplet, and so SUSY is still preserved. But the SUSY Charge has been suppressed in this Un-Chiral Multiplet Quantum Field theory.

It is clear that this is not a representation of the SUSY algebra on the physical states.

This is the kind of SUSY multiplet that we will use for the Quarks and Leptons in [34], except that we need to have the Dirac version for that. A similar exercise to the above would show that there is only a massive Fermion left in the Dirac Unchiral Multiplet, if one adds a Dirac type mass term to Section 36.

22. A Derivation of the Master Equation for the Interacting Chiral Multiplet: Let us recall the derivation of the Master Equation for the Chiral Multiplet interacting with SUSY Yang-Mills theory. Some examples of these are in [34] and the notation here is adapted to the (1,0) Multiplet used there, but we do not need much detail for this. Let us start with a Chiral Multiplet Action like that in Equation 35, except with Gauge interactions and perhaps more Chiral Multiplets too. To derive the Master Equation identity, one starts with an Action

$$\mathcal{A}_{\text{CM}}[\phi, \bar{\phi}, S, \bar{S}, Z, \bar{Z}, \Gamma, \bar{\Gamma}; \text{Gauge Fields \& Gauge Zinn Sources}] \quad (49)$$

and writes the Feynman path integral over all the fields, with Sources $j_S, \bar{j}_S, j_{\phi}, \bar{j}_{\phi}$ for the Fields and ‘Zinn Sources’ for the variations of the Fields:

$$\begin{aligned} \mathcal{G}_{\text{CM}}[Z, \bar{Z}, \Gamma, \bar{\Gamma}, j_S, \bar{j}_S, j_{\phi}, \bar{j}_{\phi}; \text{Gauge Terms}] \\ = \int \delta\phi \delta\bar{\phi} \delta S \delta\bar{S}; \delta\text{Gauge } e^{i\{\mathcal{A}_{\text{CM}} + \int d^4x (j_S S + j_{\phi} \phi + \dots + \text{Gauge Terms})\}} \end{aligned} \quad (50)$$

So we have Scalar Fields S and Fermion Fields ϕ here, and also Gauge field integration variables with a form like:

$$\delta(\text{Gauge}) = \delta\chi \delta\bar{\chi} \delta V \delta D \delta\omega \delta\zeta \quad (51)$$

where the variables are Gauge and Ghost Fields, as defined in [15]. Then one shifts all the fields as follows (λ is an anticommuting parameter so that the Grassmann nature of the fields is not violated.)

$$S \rightarrow S + \lambda \frac{\delta \mathcal{A}_{\text{CM}}}{\delta \Gamma}; \quad \phi \rightarrow \phi + \lambda \frac{\delta \mathcal{A}_{\text{CM}}}{\delta Z} \quad (52)$$

plus Complex Conjugates, and with similar terms for the Gauge Fields. We assume that the F type auxiliaries have been integrated, as in Theorem 1 above. Now since, for the Chiral Theory, we have

$$\delta_{\text{CM Fields}} \mathcal{A}_{\text{CM}} = 0 \quad (53)$$

where

$$\delta_{\text{CM Fields}} = \int d^4x \left\{ \frac{\delta \mathcal{A}_{\text{CM}}}{\delta Z^{\dot{\alpha}}} \frac{\delta}{\delta \phi_{\dot{\alpha}}} + \frac{\delta \mathcal{A}_{\text{uCM}}}{\delta \bar{Z}^{\alpha}} \frac{\delta}{\delta \bar{\phi}_{\alpha}} + \frac{\delta \mathcal{A}_{\text{uCM}}}{\delta \Gamma} \frac{\delta}{\delta S} + \frac{\delta \mathcal{A}_{\text{uCM}}}{\delta \bar{\Gamma}} \frac{\delta}{\delta \bar{S}} \right\} + \delta_{\text{Gauge Fields}} + C \bar{C} \frac{\partial}{\partial \xi} \quad (54)$$

this yields

$$\begin{aligned}
& \mathcal{G}_{\text{CM}}[Z, \bar{Z}, \Gamma, \bar{\Gamma}, j_S, \bar{j}_S, j_\phi, \bar{j}_\phi; \text{Gauge Terms}] = \\
& \int \delta\phi \delta\bar{\phi} \delta S \delta \bar{S} \delta \text{Gauge} e^{i\{\mathcal{A}_{\text{CM}} + \int d^4x (j_S S + j_\phi \phi + *)\}} \int d^4y \{(j_S \delta S - j_\phi \delta \phi + * + \text{Gauge Terms})\} \\
& \equiv \left\langle \int d^4y \{(j_S \delta S - j_\phi \delta \phi + * + \text{Gauge Terms})\} \right\rangle = 0
\end{aligned} \tag{55}$$

We follow the derivation of Zinn-Justin [28] by defining the Generator of Connected Diagrams:

$$\mathcal{G}_{\text{CM}} = e^{i\mathcal{G}_{\text{Connected}}} \tag{56}$$

and the Generator of One-Particle-Irreducible Diagrams:

$$\mathcal{G}_{\text{Connected}} = \mathcal{G}_{\text{1PI}} + \int d^4y \{(j_S S - j_\phi \phi + * + \text{Gauge Terms})\} \tag{57}$$

where the Legendre transform is defined by:

$$\frac{\delta \mathcal{G}_{\text{1PI}}}{\delta S} = j_S; \quad \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \phi} = -j_\phi \tag{58}$$

and the Zinn Sources bring in variations

$$\frac{\delta \mathcal{G}_{\text{Conn}}}{\delta \Gamma} = \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \Gamma} \equiv \langle \delta S \rangle \tag{59}$$

$$\frac{\delta \mathcal{G}_{\text{Conn}}}{\delta Z} = \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta Z} \equiv \langle \delta \phi \rangle \tag{60}$$

and their Complex Conjugates plus Gauge Terms. So we get¹⁵ the Master Equation.

$$\begin{aligned}
\mathcal{P}_{\text{CM}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta Z^{\dot{\alpha}}} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \phi_{\dot{\alpha}}} + \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{Z}^{\alpha}} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{\phi}_{\alpha}} + \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \Gamma} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta S} \right. \\
\left. + \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{\Gamma}} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{S}} \right\} + \text{Gauge Part} + \frac{\partial \mathcal{G}_{\text{1PI}}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{G}_{\text{1PI}}}{\partial \xi^{\alpha\dot{\alpha}}} = 0
\end{aligned} \tag{61}$$

which summarizes all the BRST identities here.

23. A Derivation of the Master Equation for the Interacting Un-Chiral Multiplet that arises from the Above Chiral Multiplet: Here we derive the identity for the Un-Chiral Multiplet. Exactly the same kind of reasoning applies to Half-Chiral Multiplets, but the notation is more complicated for that case. This is similar to the derivation in Section 22, but it has one step that is not immediately obvious. The Un-Chiral Multiplet is defined by simply changing variables in the Chiral Multiplet in Section 22:

$$\begin{aligned}
& \mathcal{A}_{UCM}[\phi, \bar{\phi}, J, \bar{J}, Z, \bar{Z}, \eta, \bar{\eta}; \text{Gauge Fields \& Gauge Zinn Sources}] \\
& = \mathcal{A}_{CM}[\phi, \bar{\phi}, S \rightarrow J, \bar{S} \rightarrow \bar{J}, Z, \bar{Z}, \Gamma \rightarrow \eta, \bar{\Gamma} \rightarrow \bar{\eta}; \text{Gauge Fields \& Gauge Zinn Sources}]
\end{aligned} \tag{62}$$

Now we want to derive a new Master Equation using this Action. We have a different set of variables in the Feynman path integral over all the fields, because S is gone and η has appeared:

$$\mathcal{G}_{UCM}[Z, \bar{Z}, j_\eta, \bar{j}_\eta, J, \bar{J}, j_\phi, \bar{j}_\phi; \text{Gauged}] = \int \delta\phi \delta\bar{\phi} \delta\eta \delta\bar{\eta}; \delta(\text{Gauge})$$

¹⁵ There is a problem with the ξ terms here, that can be solved by coupling to Supergravity as explained in footnote 7 on page 5.

$$e^{i\{\mathcal{A}_{\text{UCM}} + \int d^4x (j_\eta \eta + j_\phi \phi + * + \text{Gauge Terms})\}} \quad (63)$$

So now the integration variables are the Scalar Antighost Fields η and Fermion Fields ϕ , and also Gauge field integration variables. Now we do the usual shift of fields

$$\eta \rightarrow \eta + \lambda \frac{\delta \mathcal{A}_{\text{UCM}}}{\delta J} \quad (64)$$

$$\phi \rightarrow \phi + \lambda \frac{\delta \mathcal{A}_{\text{UCM}}}{\delta Z} \quad (65)$$

plus Complex Conjugates with similar terms for the Gauge Fields. Now for the moment for the Un-Chiral Multiplet, we assume the following:

$$\delta_{\text{Fields}} \mathcal{A}_{\text{UCM}} = 0 \quad (66)$$

This is the crucial point, and we will return to this below in Section 24. Now we simply shift all the fields in the integrand, with the result

$$\begin{aligned} & \int \delta\phi \delta\bar{\phi} \delta\eta \delta\bar{\eta}; \delta(\text{Gauge Fields}) e^{i\{\mathcal{A}_{\text{UCM}} + \int d^4x (j_\eta \eta + j_\phi \phi + * + \text{Gauge Terms})\}} \\ & \int d^4y \{(j_\eta \delta\eta + j_\phi \delta\phi + * + \text{Gauge Terms})\} = 0 \end{aligned} \quad (67)$$

and defining as usual

$$\mathcal{G}_{\text{UCM}} = e^{i\mathcal{G}_{\text{Connected}}} \quad (68)$$

and

$$\mathcal{G}_{\text{Connected}} = \mathcal{G}_{\text{1PI}} + \int d^4y \{(j_\eta \eta + j_\phi \phi + * + \text{Gauge Terms})\} \quad (69)$$

where

$$\frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \eta} = -j_\eta; \quad \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \phi} = -j_\phi \quad (70)$$

and their Complex Conjugates, plus Gauge Terms, we get¹⁶ the Master Equation just as we did above in Section 22:

$$\begin{aligned} \mathcal{P}_{\text{UCM}}[\mathcal{G}_{\text{1PI}}] &= \int d^4x \left\{ \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta Z^\alpha} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \phi_\alpha} + \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{Z}^\alpha} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{\phi}_\alpha} \right. \\ & \left. + \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \eta} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta J} + \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{\eta}} \frac{\delta \mathcal{G}_{\text{1PI}}}{\delta \bar{J}} \right\} + \text{Gauge Part} + \frac{\partial \mathcal{G}_{\text{1PI}}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{G}_{\text{1PI}}}{\partial \xi^{\alpha\dot{\alpha}}} = 0 \end{aligned} \quad (71)$$

This shows that this Action also generates a supersymmetric theory, although there are plenty of questions that we need to understand about the nature of that theory.

24. The Invariance Equation $\delta_{\text{Fields}} \mathcal{A}_{\text{UCM}} = 0$: The only step that was tricky in the above was Equation (66)

$$\delta_{\text{Fields}} \mathcal{A}_{\text{UCM}} = 0 \quad (72)$$

This is important, because if we needed to also vary the Zinn Sources in the Action \mathcal{A}_{UCM} , we could not derive the Master Equation, because of course Zinn Sources are not integrated in the Feynman path integral. How can we know this with so little detail being shown here?

We know from Section 22 that;

$$\mathcal{P}_{\text{CM}}[\mathcal{A}_{\text{CM}}] = \int d^4x \left\{ \frac{\delta \mathcal{A}_{\text{CM}}}{\delta Z^\alpha} \frac{\delta \mathcal{A}_{\text{CM}}}{\delta \phi_\alpha} + \frac{\delta \mathcal{A}_{\text{CM}}}{\delta \bar{Z}^\alpha} \frac{\delta \mathcal{A}_{\text{CM}}}{\delta \bar{\phi}_\alpha} \right.$$

¹⁶The same problem with the ξ terms that was mentioned in footnote 15 on page 13 occurs here again, and again it can be solved by coupling Supergravity to these new Multiplets as explained in footnote 7 on page 5

$$+\frac{\delta\mathcal{A}_{\text{CM}}}{\delta\Gamma}\frac{\delta\mathcal{A}_{\text{CM}}}{\delta S}+\frac{\delta\mathcal{A}_{\text{CM}}}{\delta\bar{\Gamma}}\frac{\delta\mathcal{A}_{\text{CM}}}{\delta\bar{S}}\Big\}+\text{Gauge Part}+\frac{\partial\mathcal{A}_{\text{CM}}}{\partial h_{\alpha\dot{\alpha}}}\frac{\partial\mathcal{A}_{\text{CM}}}{\partial\xi^{\alpha\dot{\alpha}}}=0 \quad (73)$$

and hence, because they are related by the Exchange Transformation:

$$\begin{aligned} \mathcal{P}_{\text{UCM}}[\mathcal{A}_{\text{UCM}}] = \int d^4x \Big\{ \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta Z^{\dot{\alpha}}}\frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\phi_{\dot{\alpha}}} + \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\bar{Z}^{\alpha}}\frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\bar{\phi}_{\alpha}} + \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\eta}\frac{\delta\mathcal{A}_{\text{UCM}}}{\delta J} \\ + \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\bar{\eta}}\frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\bar{J}} \Big\} + \text{Gauge Part} + \frac{\partial\mathcal{A}_{\text{UCM}}}{\partial h_{\alpha\dot{\alpha}}}\frac{\partial\mathcal{A}_{\text{UCM}}}{\partial\xi^{\alpha\dot{\alpha}}} = 0 \end{aligned} \quad (74)$$

But the latter can be written in the form:

$$\delta_{\text{UCM Fields}}\mathcal{A}_{\text{UCM}} = 0 \quad (75)$$

where

$$\begin{aligned} \delta_{\text{UCM Fields}} = \int d^4x \Big\{ \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta Z^{\dot{\alpha}}}\frac{\delta}{\delta\phi_{\dot{\alpha}}} + \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\bar{Z}^{\alpha}}\frac{\delta}{\delta\bar{\phi}_{\alpha}} + \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta J}\frac{\delta}{\delta\eta} + \frac{\delta\mathcal{A}_{\text{UCM}}}{\delta\bar{J}}\frac{\delta}{\delta\bar{\eta}} \Big\} \\ + \delta_{\text{Gauge Fields}} + C\bar{C}\frac{\partial}{\partial\xi} \end{aligned} \quad (76)$$

The point is that one can view the equation (75) either as a variation of the fields or the Zinn Sources or any combination, but there is no need to include both. So the fact that the original Action in Equation (49) is invariant under transformations of the fields alone, means that the derived Action in Equation (62) is invariant under transformations of the fields alone, even though they are different fields in the two cases. Exactly the same kind of reasoning applies to Half-Chiral Multiplets.

25. Remarks about the Hamiltonian and the Cohomology: In [39], for example, it was shown that the Hamiltonian is the sum of the SUSY charges $H = \sum Q_i^2$ and that the conservation of the SUSY charge implies that $\dot{Q}_i = [H, Q_i] = 0$. These imply that there is a set of states with the same mass and different spins. So how can these ‘Suppressed SUSY Charge’ theories exist without this structure?

The present mechanism gets around this reasoning because it is simply impossible to construct a SUSY charge Q_i in these new theories. Nevertheless, a large amount of the power of SUSY is maintained by the fact that the new Master Equation is true for the new theories. Furthermore, the algebra of the Chiral SUSY transformations (acting on the Zinn Sources as well as the Fields, in the new theories) is still used to satisfy the new Master Equation at the Lagrangian level.

It appears to be, more or less, a coincidence that the Chiral Action and the Half-Chiral Action are related by cohomology considerations¹⁷. These cohomology considerations come from the nilpotent BRST cohomology operator δ_{BRST} which is the ‘square root’ of the Master Equation. There might be a deeper connection and a clearer explanation, but, if those explanations exist, they are presently quite obscure to the author.

It is much easier to simply use the mechanism of ‘Suppressed SUSY Charge’, by noting that the SUSY charge is not present in some parts of the new actions, and that the new actions are constrained by the new Master Equation, than it is to explain ‘why’ this mechanism exists. It just does exist, and it is simple to use, and its use solves many outstanding problems of the SSM, as will be seen in [34].

26. Conclusion: In this paper we have shown that, for any chosen Chiral Multiplet, one can easily derive and write down three more theories¹⁸. The chosen Chiral Multiplet can be coupled to SUSY Gauge theory and other Chiral Multiplets, and the interactions of the new theories follow directly and simply from those interactions. But the three new Actions are quite different in their behaviour from the original chosen one. This can be done for each Chiral Multiplet in the theory independently.

¹⁷This is rather like the derivation of the new transformations in [15] from the notion of ‘BRST Recycling’—there are a limited number of ways to construct these actions, and so things ‘pop up again’ in unexpected places.

¹⁸For a Dirac Multiplet this is limited by the conserved global U(1) phase, which must be conserved.

It is in the Chiral form that it is easy to write down the couplings. Then one simply implements the appropriate set of Exchange Transformations, which result in a new Action and a new Master Equation. The new Action yields zero for the new Master Equation. This Exchange Transformation takes all or part of the Scalar Fields S and replaces them with Zinn Sources J , and also takes the related Zinn Sources Γ and replaces them with Antighost Fields η . These Exchange Transformations are closely related to Canonical Transformations, as is explained in Section 15.

These new Actions were discovered by generating the Irreducible Chiral Dotted Spinor Superfield in [15], in the hope of coupling the BRST cohomology of the Chiral Multiplet to it. Once the auxiliary W was integrated in that theory, it was noticed that the resulting Half-Chiral Action could be generated by an Exchange Transformation from a Chiral Action that has its auxiliary F integrated. So, in a sense, that coupling of the cohomology has now been done, and the result is that we have discovered new ways to realize SUSY in local Actions, and those new Half-Chiral Actions can be coupled to Gauge theory and each other (and even Supergravity).

But the result here goes farther, because we also have discovered Un-Chiral Multiplets that do not arise by way of the Irreducible Chiral Dotted Spinor Superfield of [15]. The Un-Chiral Multiplet Action arises simply by taking the full version of the Exchange Transformation that was suggested by the existence of the Half-Chiral Multiplet.

Supercharges are not constructible in the new theories (except in some sectors), because when the Zinn Sources become involved, no divergenceless SUSY current exists. This happens because the Zinn sources are not quantized, and they do not satisfy Equations of Motion.

The new theory has new Fields and Sources, and so it has a different generating functional for the Feynman path integral, as shown in Sections 22 and 23. But because the Actions and Master Equations are related through an Exchange Transformation, it was easy to prove, in Sections 23 and 24 that the new Action satisfies the new Master Equation for its 1PI Generating Functionals \mathcal{G}_{1PI} .

In Sections 16, 17 and 18 we set out the details for the massive Half-Chiral Multiplet case, which shows explicitly how the effect of a SUSY Charge acts like it is ‘half-present’. In Section 19 we observed the interactions of the Half-Chiral Multiplet.

Section 21 discusses mass for the Un-Chiral Multiplet case and shows that the SUSY Charge is completely gone there. In that case we need to couple the theory to SUSY Gauge Theory to get an interaction.

The result is that that these new theories, when calculated in the renormalized Feynman expansion, iteratively, loop by loop, should be as valid as is the original purely Chiral theory [40]. But the theories are very different, of course, and the Half-Chiral and Un-Chiral theories are not subject to the SUSY algebra, as described in [36], because they do not have conserved SUSY Charges. They are lacking in Scalar Fields compared to Chiral Multiplets. We shall use some of these new Half-Chiral Multiplets and Un-Chiral Multiplets, coupled to SUSY Gauge theory and to each other, to write down a new kind of SSM, in [34].

In the Appendix at Section 27, we review the Dirac case. This is necessary to describe Leptons and Quarks, since they have conserved Baryon and Lepton numbers. This is not very different from the Majorana case which is discussed above.

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Appendix: The Dirac Irreducible Chiral Dotted Spinor Superfield, and the Dirac Half-Chiral Multiplet

27. The Dirac Irreducible Chiral Dotted Spinor Superfield: The Majorana Irreducible Chiral Dotted Spinor Supermultiplet treated above is the Majorana version of the work in [15,16,17]. The progress reported there was to find and use the following free massless kinetic Action for a generic Dirac (ie Complex) Irreducible Chiral Dotted Spinor Superfield¹⁹:

$$\mathcal{A}_{\text{DI}} = \mathcal{A}_{\text{DI Kinetic}} + \mathcal{A}_{\text{DI Zinn}} + \mathcal{A}_{\text{SUSY}} \quad (77)$$

28 The Dirac Irreducible Chiral Dotted Spinor Superfield has the following kinetic Action. We will use the notation of [15]. Here we also will add a group representation²⁰ index i :

$$\begin{aligned} \mathcal{A}_{\text{DI Kinetic}} = \int d^4x \left\{ -\phi_L^{i\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}_{Li}^\alpha - \phi_{Ri}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}_R^{i\alpha} - W_{\alpha\dot{\alpha}}^i \bar{W}_i^{\alpha\dot{\alpha}} + H^i \square \bar{H}_i \right. \\ \left. - \frac{\sqrt{2}}{2} \bar{\eta}_i \left(\phi_L^{i\dot{\delta}} \bar{C}_{\dot{\delta}} + \bar{\phi}_R^{i\delta} C_\delta \right) - \frac{\sqrt{2}}{2} \eta^i \left(\bar{\phi}_{Li}^\delta C_\delta + \phi_{Ri}^{\dot{\delta}} \bar{C}_{\dot{\delta}} \right) \right\} \end{aligned} \quad (78)$$

29. Here is the Zinn Action that goes with this Action, also taken from [15], with indices added.

$$\begin{aligned} \mathcal{A}_{\text{DI Zinn}} = \int d^4x Z_{Ri}^{\dot{\alpha}} \left(-\frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} H^i C^\alpha - W_{\alpha\dot{\alpha}}^i C^\alpha + \xi \cdot \partial \phi_{Li}^\alpha \right) \\ + Z_L^{i\dot{\alpha}} \left(\frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} \bar{H}_i C^\alpha - \bar{W}_{i\alpha\dot{\alpha}} C^\alpha + \xi \cdot \partial \phi_{Ri}^\alpha \right) + \Sigma^{i\alpha\dot{\alpha}} \left(\sqrt{2} \bar{\eta}_i \bar{C}_{\dot{\alpha}} C_\alpha - \frac{1}{2} \partial_\alpha^{\dot{\gamma}} \phi_{Ri\dot{\gamma}} \bar{C}_{\dot{\alpha}} \right. \\ \left. - \frac{1}{2} \partial_\alpha^{\dot{\gamma}} \phi_{Ri\dot{\alpha}} \bar{C}_{\dot{\gamma}} - \frac{1}{2} \partial_\alpha^{\dot{\gamma}} \bar{\phi}_{Li\dot{\gamma}} C_\alpha - \frac{1}{2} \partial_\alpha^{\dot{\gamma}} \bar{\phi}_{Li\dot{\alpha}} C_\gamma + \xi \cdot \partial \bar{W}_{i\alpha\dot{\alpha}} \right) \\ + \bar{\Upsilon}_i \left(\frac{1}{\sqrt{2}} \bar{\phi}_{R\beta}^i C^\beta - \frac{1}{\sqrt{2}} \phi_{L\beta}^i \bar{C}^{\dot{\beta}} + \xi \cdot \partial G^i \right) + \frac{1}{\sqrt{2}} J^i \left(\partial_{\gamma\dot{\delta}} \bar{W}_i^{\gamma\dot{\delta}} + \xi \cdot \partial \bar{\eta}_i \right) + * + \mathcal{A}_{\text{SUSY}} \end{aligned} \quad (79)$$

30. Here is the Master Equation that goes with this Action, also taken from [15], with indices added.

$$\begin{aligned} \mathcal{P}_{\text{DI}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z_L^{i\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{Ri\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta Z_{Ri}^{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{Li}^\alpha} + \frac{\delta \mathcal{A}}{\delta \bar{\Sigma}_i^{\alpha\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta W_{\alpha\dot{\alpha}}^i} \right. \\ \left. + \frac{\delta \mathcal{A}}{\delta \bar{\Upsilon}_i} \frac{\delta \mathcal{A}}{\delta H^i} + \frac{\delta \mathcal{A}}{\delta \bar{J}_i} \frac{\delta \mathcal{A}}{\delta \eta^i} \right\} + * + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \end{aligned} \quad (80)$$

¹⁹Go to reference [15], and change $E \rightarrow \sqrt{2}G$, $\bar{\Upsilon} \rightarrow \frac{1}{\sqrt{2}}\bar{\Upsilon}$ and $\eta \rightarrow \sqrt{2}\eta$, $J \rightarrow \frac{1}{\sqrt{2}}J$ and change overall sign of kinetic term.

²⁰Here is the rule governing the index: *Index Rule: All the left objects have index up, and all the right objects have the index down. The reverse rule holds for the Complex Conjugate, and the Complex Conjugate of left is right and vice versa. In special cases there may also be a way to raise and lower the index with some invariant tensor.*

31. Integrate the Auxiliary W to form the Half-Chiral Multiplet Action: There is little difference between this Dirac version and the Majorana version analyzed above, except that the Dirac version can admit a conserved quantum number like Lepton number. After integrating the auxiliary out, the resulting Action can be written in the form:

$$\begin{aligned}
\mathcal{A}_{\text{DHC}} = \int d^4x \left\{ -\phi_L^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}_L^{\alpha} - \phi_R^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\phi}_R^{\alpha} + \frac{1}{2} \partial_{\alpha\dot{\alpha}} H \partial^{\alpha\dot{\alpha}} \bar{H} + \frac{1}{2} \partial_{\alpha\dot{\alpha}} J \partial^{\alpha\dot{\alpha}} \bar{J} \right. \\
+ \bar{\eta} \left(\frac{1}{\sqrt{2}} \phi_{L\dot{\delta}} \bar{C}^{\dot{\delta}} + \frac{1}{\sqrt{2}} \bar{\phi}_{R\dot{\delta}} C^{\dot{\delta}} \right) + \eta \left(\frac{1}{\sqrt{2}} \bar{\phi}_{L\dot{\delta}} C^{\dot{\delta}} + \frac{1}{\sqrt{2}} \phi_{R\dot{\delta}} \bar{C}^{\dot{\delta}} \right) \\
+ \Upsilon \left(\frac{1}{\sqrt{2}} \bar{\phi}_{R\beta} C^{\beta} - \frac{1}{\sqrt{2}} \phi_{L\beta} \bar{C}^{\beta} \right) + \Upsilon \left(\frac{1}{\sqrt{2}} \phi_{R\beta} \bar{C}^{\beta} - \frac{1}{\sqrt{2}} \bar{\phi}_{L\beta} C^{\beta} \right) \\
+ \bar{Z}_{R\alpha} \bar{C}_{\dot{\alpha}} \left(-\frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} \bar{H} + \frac{1}{\sqrt{2}} \partial^{\alpha\dot{\alpha}} \bar{J} \right) + \bar{Z}_L \bar{C}^{\dot{\alpha}} \left(\frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} H + \frac{1}{\sqrt{2}} \partial^{\alpha\dot{\alpha}} J \right) \\
+ Z_R^{\dot{\alpha}} C^{\alpha} \left(-\frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} H + \frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} J \right) + Z_L^{\dot{\alpha}} C^{\alpha} \left(\frac{1}{\sqrt{2}} \partial_{\alpha\dot{\alpha}} \bar{H} + \frac{1}{\sqrt{2}} \partial^{\alpha\dot{\alpha}} \bar{J} \right) \\
+ Z_{L\dot{\alpha}} C_{\alpha} \bar{Z}_L^{\alpha} \bar{C}^{\dot{\alpha}} + \bar{Z}_{R\alpha} \bar{C}_{\dot{\alpha}} Z_R^{\dot{\alpha}} C^{\alpha} + (Z_R \xi \cdot \partial \phi_L + Z_L \xi \cdot \partial \phi_R + \Upsilon \xi \cdot \partial H + J \xi \cdot \partial \eta + *) \quad (81)
\end{aligned}$$

32. Master Equation for Half-Chiral Multiplet: We temporarily dropped the indices i in the above and now we will restore them to their proper places. The Action above in Section 31 yields zero²¹ for the following Master Equation²²:

$$\mathcal{P}_{\text{DHC}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z_L^{i\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{Ri\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta Z_R^{i\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{Li\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \bar{\Upsilon}_i} \frac{\delta \mathcal{A}}{\delta H^i} + \frac{\delta \mathcal{A}}{\delta \bar{J}_i} \frac{\delta \mathcal{A}}{\delta \eta^i} + * \right\} + \frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}} \quad (82)$$

33. Remarkable Symmetries of the Action \mathcal{A}_{DHC} : As was remarked for the Majorana case in Section 31 above, the above Dirac type Action \mathcal{A}_{DHC} has a remarkable symmetry which was not obvious before we integrated the auxiliary W . The Field η and the Source Υ appear in similar ways to each other. The Field H and the Source J also appear in similar ways to each other. The term $\frac{1}{2} \partial_{\alpha\dot{\alpha}} J \partial^{\alpha\dot{\alpha}} \bar{J}$ looks like a kinetic term for J , except that J is a Source. The term $\eta \left(\frac{1}{\sqrt{2}} \bar{\phi}_{L\dot{\delta}} C^{\dot{\delta}} + \frac{1}{\sqrt{2}} \phi_{R\dot{\delta}} \bar{C}^{\dot{\delta}} \right)$ looks like a Zinn Source coupled to a variation, except that η is a Field.

34. This symmetry can be exploited with the following Generator for an Exchange Transformation of the Action which will leave the Master Equation invariant, but of a new form. Our new Action will have ‘new Fields’ (H_L, H_R) and ‘new Sources’ (Γ_L, Γ_R). These will replace the ‘old Fields’ (H, η) and the ‘old Zinn Sources’ (J, Υ). We choose a generating function \mathcal{G}_{DHC} of the new Zinn Sources (Γ_L, Γ_R) and the old Field H and the old Zinn Source J :

$$\mathcal{G}_{\text{DHC}} = \int d^4x \left\{ \frac{1}{\sqrt{2}} (\bar{H}_i + \bar{J}_i) \Gamma_L^i + \frac{1}{\sqrt{2}} (J^i - H^i) \Gamma_{Ri} + \frac{1}{\sqrt{2}} (H^i + J^i) \bar{\Gamma}_{Li} + \frac{1}{\sqrt{2}} (\bar{J}_i - \bar{H}_i) \bar{\Gamma}_R^i \right\} \quad (83)$$

35. Equations from the Generator and their inverses Now using the Generator defined in Section 34 above, we get

²¹We have dropped the Source $\Sigma_{\alpha\dot{\alpha}}$ for the auxiliary Field here of course. This is an application of the theorem in Section 8.

²²We have simply removed the terms $\frac{\delta \mathcal{A}}{\delta \Sigma_{\alpha\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta W_{\alpha\dot{\alpha}}^i}$ from 80 in accord with the theorem in Section 8.

$$\begin{aligned}
H_L^i &= \frac{\delta \mathcal{G}}{\delta \Gamma_{Ri}} = \frac{1}{\sqrt{2}} (J^i - H^i); \quad H_{Ri} = \frac{\delta \mathcal{G}}{\delta \Gamma_L^i} = \frac{1}{\sqrt{2}} (\overline{H}_i + \overline{J}_i) \\
\overline{\eta}_i &= \frac{\delta \mathcal{G}}{\delta J^i} = \frac{1}{\sqrt{2}} (\Gamma_{Ri} + \overline{\Gamma}_{Li}); \quad \overline{\Upsilon}_i = \frac{\delta \mathcal{G}}{\delta G^i} = \frac{1}{\sqrt{2}} (-\Gamma_{Ri} + \overline{\Gamma}_{Li}) \\
\Gamma_{Ri} &= \frac{1}{\sqrt{2}} (-\overline{\Upsilon}_i + \overline{\eta}_i); \quad \Gamma_L^i = \frac{1}{\sqrt{2}} (\Upsilon^i + \eta^i); \\
\overline{J}_i &= \frac{1}{\sqrt{2}} (H_{Ri} + \overline{H}_{Li}); \quad \overline{H}_i = \frac{1}{\sqrt{2}} (H_{Ri} - \overline{H}_{Li})
\end{aligned} \tag{84}$$

36. Chiral Action for Dirac Multiplet Using the above definitions, the Action can be written in terms of the new variables as follows:

$$\begin{aligned}
\mathcal{A}_{\text{DCL}} &= \int d^4x \left\{ -\phi_L^{i\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \overline{\phi}_{Li}^\alpha + \frac{1}{2} \partial^{\alpha\dot{\alpha}} \overline{H}_{Li} \partial_{\alpha\dot{\alpha}} H_L^i + \Gamma_{Ri} \phi_{L\dot{\delta}}^i \overline{C}^{\dot{\delta}} + \overline{\Gamma}_R^i \overline{\phi}_{Li\dot{\delta}} C^{\dot{\delta}} \right. \\
&\quad \left. + Z_{Ri}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} H_L^i C^\alpha + \overline{Z}_R^{i\alpha} \partial_{\alpha\dot{\alpha}} \overline{H}_{Li} C^\alpha + Z_{Ri\dot{\alpha}} C_\alpha \overline{Z}_R^{\alpha\dot{\alpha}} \overline{C}^{\dot{\alpha}} + \xi \text{ terms} \right\}
\end{aligned} \tag{85}$$

plus a similar term with $L \rightarrow R$:

$$\begin{aligned}
\mathcal{A}_{\text{DCR}} &= \int d^4x \left\{ -\phi_{Ri}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \overline{\phi}_R^{i\alpha} + \frac{1}{2} \partial^{\alpha\dot{\alpha}} \overline{H}_R^i \partial_{\alpha\dot{\alpha}} H_{Ri} + \Gamma_L^i \phi_{Ri\dot{\delta}} \overline{C}^{\dot{\delta}} + \overline{\Gamma}_{Li} \overline{\phi}_{R\dot{\delta}}^i C^{\dot{\delta}} \right. \\
&\quad \left. + Z_L^{i\dot{\alpha}} \partial_{\alpha\dot{\alpha}} H_{Ri} C^\alpha + \overline{Z}_{Li}^{\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \overline{H}_R^i C^\alpha + Z_{L\dot{\alpha}}^i C_\alpha \overline{Z}_{Li}^{\alpha\dot{\alpha}} \overline{C}^{\dot{\alpha}} + \xi \text{ terms} \right\}
\end{aligned} \tag{86}$$

We must add $+\mathcal{A}_{\text{SUSY}}$ as usual to the above. This is a pair of Chiral Multiplets that we can put together to make a Dirac Chiral Multiplet.

37. New Form of Master Equation In terms of the new Fields, the Master Equation takes the following ‘separated’ form.

$$\mathcal{P}_{\text{DCL}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z_{Ri}^{\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{L\dot{\alpha}}^i} + \frac{\delta \mathcal{A}}{\delta \Gamma_{Ri}} \frac{\delta \mathcal{A}}{\delta H_L^i} + \frac{\delta \mathcal{A}}{\delta \overline{Z}_R^{i\alpha}} \frac{\delta \mathcal{A}}{\delta \overline{\phi}_{Li\alpha}} + \frac{\delta \mathcal{A}}{\delta \overline{\Gamma}_R^i} \frac{\delta \mathcal{A}}{\delta \overline{H}_{Li}} \right\} \tag{87}$$

plus

$$\mathcal{P}_{\text{DCR}}[\mathcal{A}] = \int d^4x \left\{ \frac{\delta \mathcal{A}}{\delta Z_L^{i\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \phi_{Ri\dot{\alpha}}} + \frac{\delta \mathcal{A}}{\delta \Gamma_L^i} \frac{\delta \mathcal{A}}{\delta H_{Ri}} + \frac{\delta \mathcal{A}}{\delta \overline{\Gamma}_{Li}} \frac{\delta \mathcal{A}}{\delta \overline{H}_R^i} + \frac{\delta \mathcal{A}}{\delta \overline{Z}_{Li}^{\alpha\dot{\alpha}}} \frac{\delta \mathcal{A}}{\delta \overline{\phi}_{R\dot{\alpha}}^i} \right\} \tag{88}$$

We must add $+\frac{\partial \mathcal{A}}{\partial h_{\alpha\dot{\alpha}}} \frac{\partial \mathcal{A}}{\partial \xi^{\alpha\dot{\alpha}}}$ as usual to the above. These add to make a Dirac Master Equation for the Chiral Multiplet pair.

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